ME 321: FLUID MECHANICS-I

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Lecture-11

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Momentum Principle

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RTT (**R**eynolds **T**ransport **T**heorem) relates between the system approach with finite control volume (CV) approach for a system property:

$$\frac{dB_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{CV}} \rho\beta \, d\mathcal{V} + \int_{\text{CS}} \rho\beta \left(\vec{\mathbf{V}} \cdot \hat{\mathbf{n}}\right) dA$$

B = any extensive property (such as mass, momentum, energy etc.)

b = any intensive property per unit mass (such as mass per mass, momentum per mass, etc.)

Conservation of linear momentum

$$B = M\vec{\mathbf{V}} \text{ (momentum)}, \qquad \beta = \frac{M\vec{\mathbf{V}} \text{ (momentum)}}{M \text{ (mass)}} = \vec{\mathbf{V}}$$

RTT takes the form of

$$\frac{d\left(M\vec{\mathbf{V}}\right)_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{CV}} \vec{\mathbf{V}}\rho \, d\mathcal{V} + \int_{\text{CS}} \vec{\mathbf{V}}\rho\left(\vec{\mathbf{V}} \cdot \hat{\mathbf{n}}\right) dA$$

Recall linear momentum equation

Newton's second law of motion for a system is

$$\frac{d\left(M\vec{\mathbf{V}}\right)_{\text{sys}}}{dt}=\sum\vec{F}_{\text{sys}}=\sum\vec{F}_{\substack{\text{contents of the}\\ \text{control volume}}}$$

For fluid dynamics:

$$\sum\vec{F}_{\substack{\text{contents of the}\\ \text{control volume}}}=\sum\left(\vec{F}_s+\vec{F}_B\right)=\frac{d}{dt}\int_{\text{CV}}\vec{\mathbf{V}}\rho\,d\forall+\int_{\text{CS}}\vec{\mathbf{V}}\rho\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right)dA$$

** Vector equation

$\sum\vec{F}$: External forces acting on the content of the control volume (CV)

(such as pressure force, viscous shear force, gravity etc.)

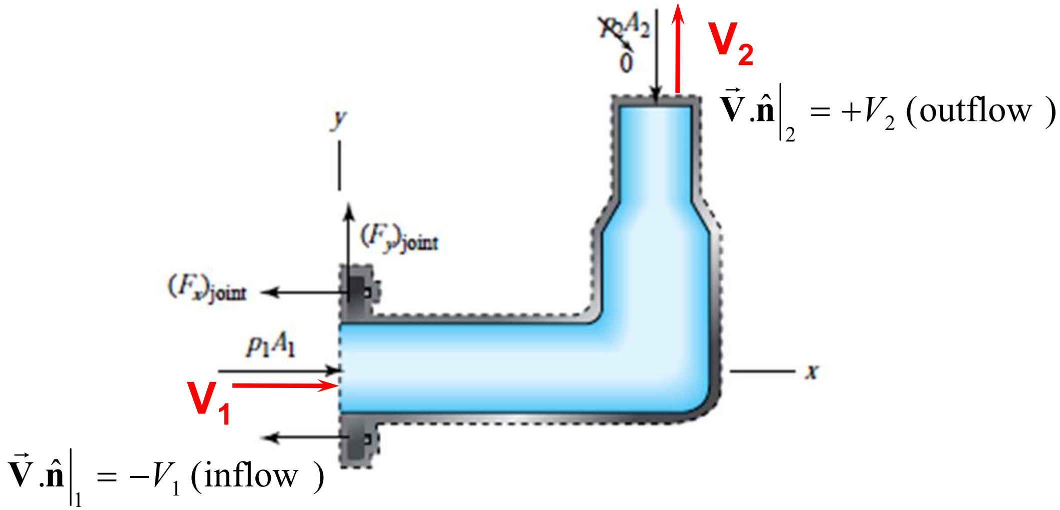
Momentum Principle

The above expression could be simplified considerably if a flow system has **only one entrance and one exit and if the flow is steady**:

$$\sum \vec{F}_{CV} = \frac{d}{dt}\int_{CV} \vec{\mathbf{V}}\rho\, d\forall + \int_{CS} \vec{\mathbf{V}}\rho\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right)dA$$

$$\Rightarrow \sum \vec{F}_{CV} = \int_{CS} \vec{\mathbf{V}}\rho\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right)dA$$

$$\Rightarrow \sum \vec{F}_{CV} = \rho_2 A_2 V_2 \vec{\mathbf{V}}_2 - \rho_1 A_1 V_1 \vec{\mathbf{V}}_1$$



Using continuity:

$$\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \qquad \text{(mass flow rate)}$$

Then:

$$\boxed{\sum \vec{F}_{CV} = \dot{m}\left(\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1\right)}$$

Note that the momentum equation is a vector equation which represents three scalar equations:

$$x: \quad \sum F_x = \dot{m}\left(V_{2x} - V_{1x}\right)$$

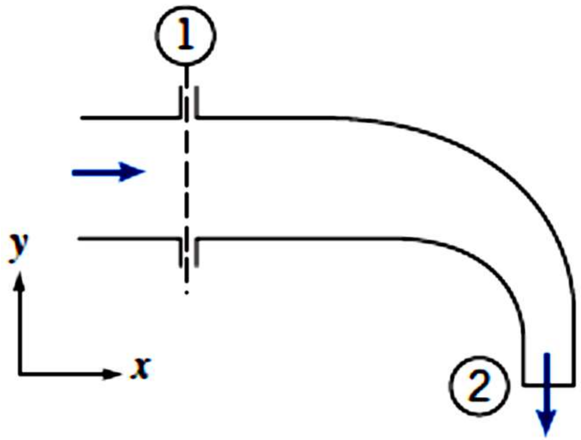
$$y: \quad \sum F_y = \dot{m}\left(V_{2y} - V_{1y}\right)$$

$$z: \quad \sum F_z = \dot{m}\left(V_{2z} - V_{1z}\right)$$

This is the fundamental principle which drives the turbomachinery (propulsion nozzle in jet engine, turbine, compressor cascade etc.)

Problem # 1

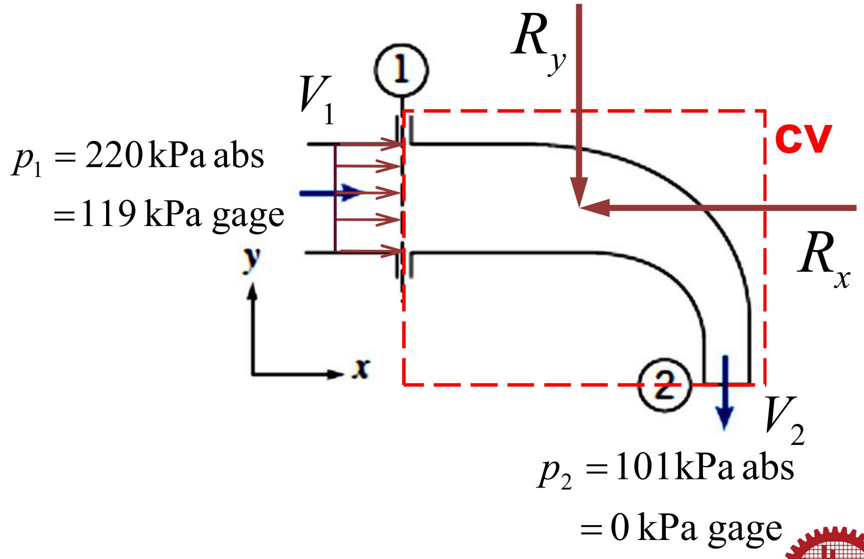
Water flows steadily through the 90° reducing elbow as shown in figure. At the inlet of the elbow, the absolute pressure is 220 kPa and the cross-sectional area is 0.01 m^2. At the outlet, the cross-sectional area is 0.0025 m^2 and the velocity is 16 m/s. The elbow discharges to the atmosphere. Determine the force required to hold the elbow in place.



Solution:

From continuity equation:

$$Q_1 = Q_2 \quad \Rightarrow A_1V_1 = A_2V_2$$
$$\Rightarrow (0.01)V_1 = (0.0025)(16)$$
$$\therefore V_1 = 4 \text{ m/s}$$



from steady flow momentum principle:

$$\sum \vec{F}_{CV} = \int_{CS} \vec{\mathbf{V}} \rho \left(\vec{\mathbf{V}} \cdot \hat{\mathbf{n}} \right) dA$$

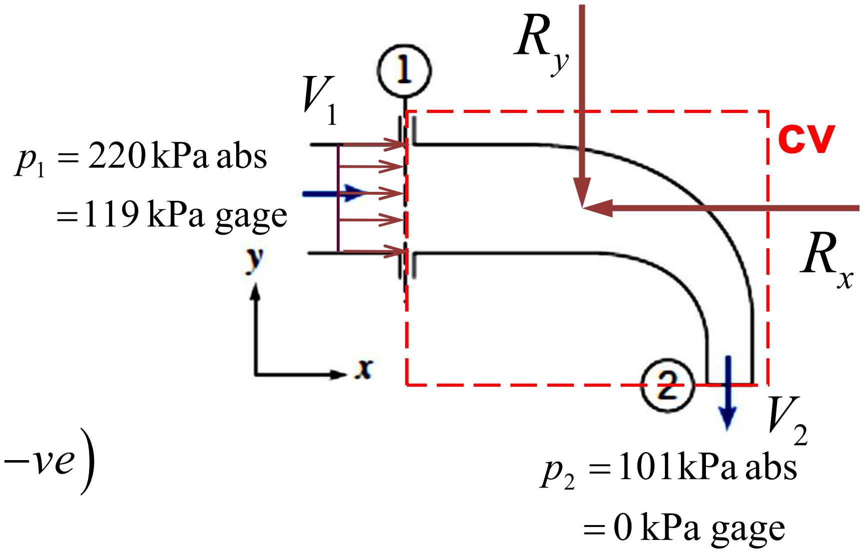
$$\rightarrow x : \sum F_x = \dot{m}\left(V_{2x} - V_{1x}\right)$$
$$\Rightarrow -R_x + p_1A_1 = \dot{m}\left(V_{2x} - V_{1x}\right) = \dot{m}\left(0 - V_{1x}\right) = -\left(\rho V_1 A_1\right)V_{1x}$$
$$\Rightarrow R_x = \left(\rho V_1 A_1\right)V_1 + p_1A_1$$

Then

$$\Rightarrow R_x = (\rho V_1 A_1)V_1 + p_1 A_1$$

$$\Rightarrow R_x = (119\times10^3)(0.01) + (1000\times4\times0.01)(4)$$

$$\therefore R_x = 1.35\,\text{kN}$$



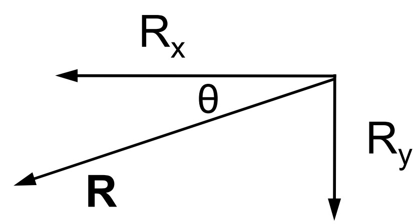
Now, along y-axis:

$$\uparrow y:\ \sum R_y = \dot m (V_{2y} - V_{1y})$$

$$\Rightarrow -R_y = \dot m (V_{2y} - V_{1y}) = \dot m(-V_{2y} - 0) = -(\rho V_2 A_2)V_2 \quad ;(V_2 \downarrow -ve)$$

$$\Rightarrow R_y = (1000\times16\times0.0025)(16)$$

$$\therefore R_y = 0.64\,\text{kN}$$



So, the magnitude of resultant force is

$$R = \sqrt{R_x^2 + R_y^2}$$

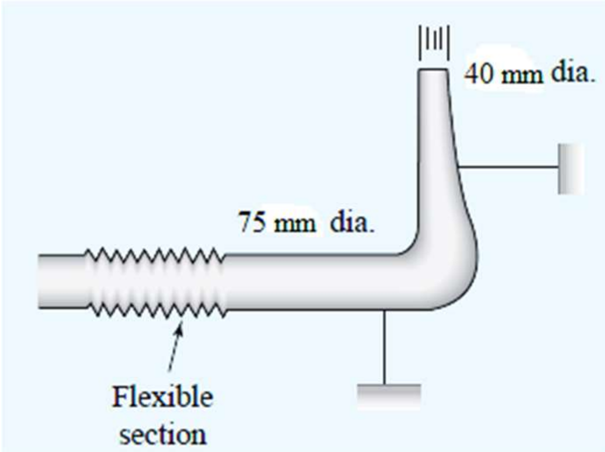
$$\Rightarrow R = \sqrt{1.35^2 + 0.64^2}$$

$$\Rightarrow R = 1.49\,\text{kN}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = 25.37^\circ \ \text{ with } (-\text{ve})\ x-\text{axis}$$

Problem # 2

Water flows through a horizontal pipe bend and exits into the atmosphere as shown in figure. The flow rate is 30 m^3/hr. Calculate the force in each of the rods holding the pipe bend in position. Neglect body forces and viscous effects and shear force in the rods.



Solution:

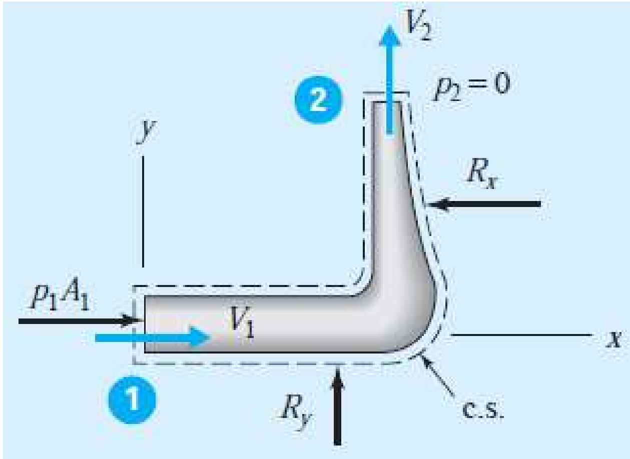
Now, from continuity equation:

$$Q_1 = Q_2 \quad \Rightarrow A_1V_1 = A_2V_2 = 30/(60\times 60)$$

$$\Rightarrow \frac{\pi}{4}\left(75\times 10^{-3}\right)^2 V_1 = \frac{\pi}{4}\left(40\times 10^{-3}\right)^2 V_2 = 8.33\times 10^{-3}$$

$$\therefore V_1 = 1.89\,\text{m/s}$$

$$V_2 = 6.63\,\text{m/s}$$



Use Bernoulli equation to determine the pressure at section 1:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$\Rightarrow \frac{p_1}{\gamma} + \frac{1.89^2}{2g} + 0 = \frac{0}{\gamma} + \frac{6.63^2}{2g} + 0$$

$$\Rightarrow p_1 = 20192.4 \quad \text{Pa}$$

Problem # 2 cont…

From steady flow momentum principle:

$$\sum \vec{F}_{CV} = \int_{CS} \vec{\mathbf{V}} \rho \left(\vec{\mathbf{V}} \cdot \hat{\mathbf{n}} \right) dA$$

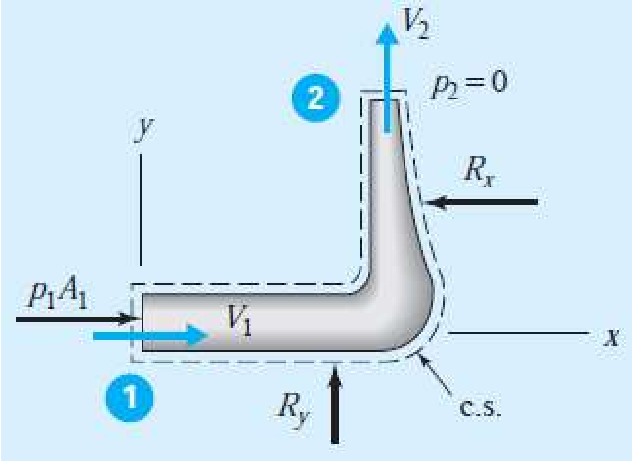
$$\rightarrow x: \quad \sum F_x = \dot{m} \left(V_{2x} - V_{1x} \right)$$

$$\Rightarrow -R_x + p_1 A_1 = \dot{m} \left(V_{2x} - V_{1x} \right) = \dot{m} \left(0 - V_{1x} \right) = -\dot{m} V_{1x}$$

$$\Rightarrow R_x = \dot{m} V_{1x} + p_1 A_1$$

$$\Rightarrow R_x = \frac{30}{(60 \times 60)} (1000)(1.89) + (20192.4) \left(\frac{\pi}{4} \left(75 \times 10^{-3} \right)^2 \right)$$

$$\therefore R_x = 104.95 \text{ N} \leftarrow$$



For y-direction:

$$\uparrow y: \quad \sum F_y = \dot{m} \left(V_{2y} - V_{1y} \right)$$

$$\Rightarrow R_y = \dot{m} \left(V_{2y} - V_{1y} \right) = \dot{m} \left(V_{2y} - 0 \right) = \dot{m} V_{2y}$$

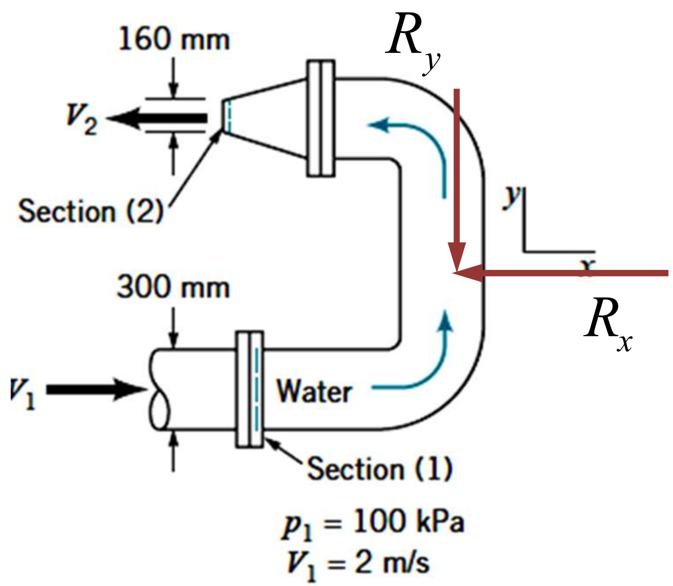
$$\Rightarrow R_y = \dot{m} V_{2y}$$

$$\Rightarrow R_y = \frac{30}{(60 \times 60)} (1000)(6.63)$$

$$\therefore R_y = 55.25 \text{ N} \leftarrow$$

Problem # 3

Determine the magnitude and direction of the anchoring force needed to hold the horizontal elbow and nozzle combination shown in figure in place. Atmospheric pressure is 100 kPa (abs) The gage pressure at section (1) is 100 kPa. At section (2), the water exits to the atmosphere.



Solution: from steady flow momentum principle;

$$\Rightarrow \sum \vec{F}_{CV} = \int_{CS} \vec{\mathbf{V}}\rho\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right)dA$$

$$\rightarrow x: \sum F_x = \dot{m}\left(V_{2x} - V_{1x}\right)$$

$$\Rightarrow -R_x + p_1 A_1 = \dot{m}\left(V_{2x} - V_{1x}\right) = \rho V_1 A_1\left(-V_2 - V_1\right) \quad ; \qquad \left(V_2 \leftarrow \; -ve\right)$$

$$\Rightarrow R_x = p_1 A_1 + \left(\rho V_1 A_1\right)\left(V_1 + V_2\right)$$

Now, from continuity equation: $Q_1 = Q_2 \quad \Rightarrow A_1 V_1 = A_2 V_2$

$$\Rightarrow \left(\frac{\pi}{4} d_1^2\right)(V_1) = \left(\frac{\pi}{4} d_2^2\right)(V_2)$$

$$\Rightarrow V_2 = \left(\frac{d_1}{d_2}\right)^2 V_1$$

$$\therefore V_2 = 7.03 \text{ m/s}$$

Then,

$$\rightarrow x: \;\Rightarrow R_x = p_1 A_1 + (\rho V_1 A_1)(V_1 + V_2)$$

$$\Rightarrow F_x = (100\times 10^3)\left(\frac{\pi}{4}\times 0.3^2\right) + \left(1000\times 2\times \frac{\pi}{4}\times 0.3^2\right)(2+7.03)$$

$$\therefore\; F_x = 8.35\,\text{kN}$$

Now, along y-axis:

$$\uparrow y: \; -R_y = \dot{m}(V_{2y} - V_{1y}) = \dot{m}(0-0) = 0 \qquad \therefore R_y = 0$$

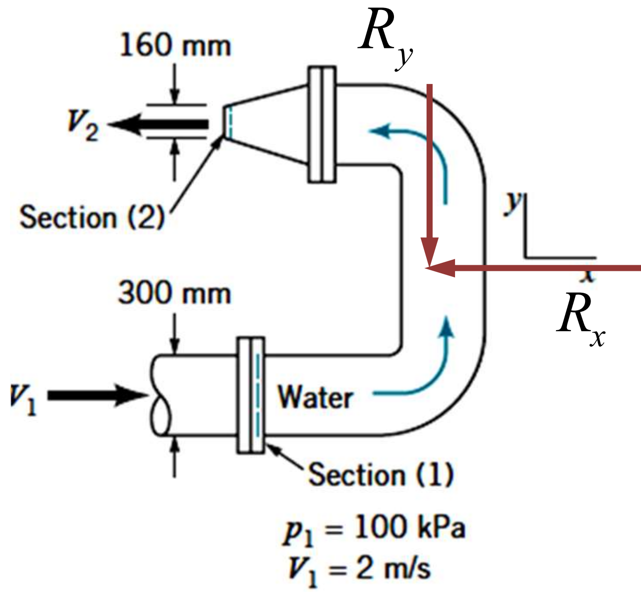
So, the magnitude of resultant anchoring force is

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\Rightarrow R = \sqrt{8.35^2 + 0^2}$$

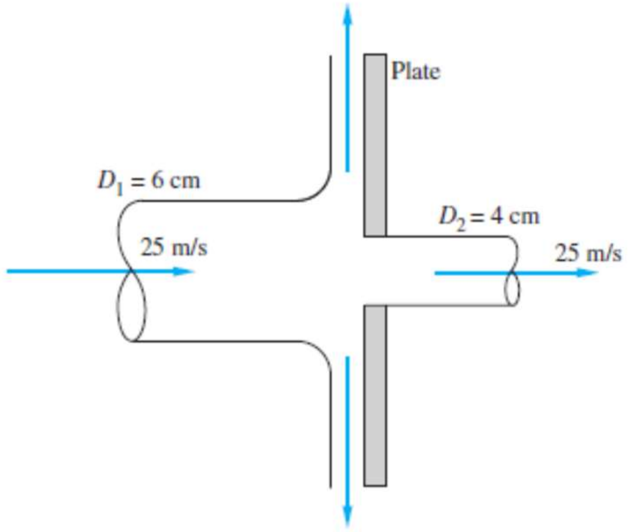
$$\Rightarrow R = 8.35\,\text{kN}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \;0^\circ \;\text{ with}(-\text{ve})\,x - \text{axis}$$



Problem 4

The 6-cm-diameter 20°C water jet strikes a plate containing a hole of 4-cm diameter. Part of the jet passes through the hole, and part is deflected. Determine the horizontal force required to hold the plate.



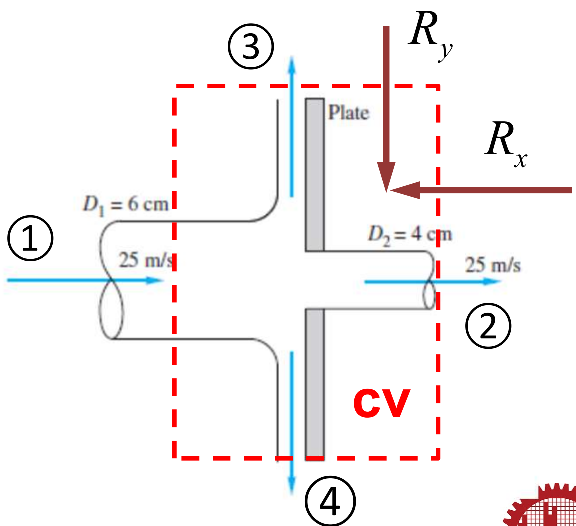
Solution:

$$Q_1 = \frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4}(0.06)^2(25) \;=\; 0.0707 \;\; \text{m}^3/\text{s}$$

$$Q_2 = \frac{\pi}{4} d_2^2 V_2 = \frac{\pi}{4}(0.04)^2(25) \;=\; 0.0314 \;\; \text{m}^3/\text{s}$$

from steady flow momentum principle:

$$\boxed{\sum \vec{F}_{CV} = \int_{CS} \vec{\mathbf{V}}\rho\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right)dA}$$



$$\rightarrow x: \sum F_x = \left(\dot{m} V_x\right)_{out} - \left(\dot{m} V_x\right)_{in}$$

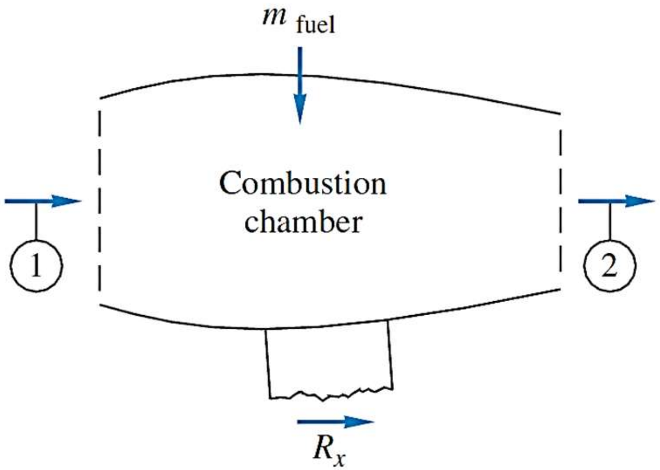
$$\Rightarrow -R_x = \left(\dot{m}_2 V_{2x} + \dot{m}_3 V_{3x} + \dot{m}_4 V_{4x}\right) - \dot{m}_1 V_{1x}$$

$$\Rightarrow -R_x = \left((1000)(0.0314)(25) + \dot{m}_3(0) + \dot{m}_4(0)\right) - (1000)(0.0707)(25)$$

$$\therefore \quad R_x = 982.5\,\text{N} \qquad \text{(to left)} \qquad \text{Ans.}$$

Problem 5

(a) The jet engine on a test stand admits air at 20°C and 1 atm at section 1, where A_1 = 0.5 m^2 and V_1 = 250 m/s. The fuel-to-air ratio is 1:30. The air leaves section 2 at atmospheric pressure and higher temperature, where V_2 = 900 m/s and A_2 = 0.4 m^2 . Compute the horizontal test stand reaction $\mathbf{R_x}$ needed to hold this engine fixed.



Solution:

$$\rho_1 = \frac{p}{RT} = \frac{101325}{(287)(273+20)} = 1.205 \text{ kg/m}^3$$

$$\dot{m}_1 = \rho_1 A_1 V_1 = (1.205)(0.5)(250) = 150.6 \text{ kg/s}$$

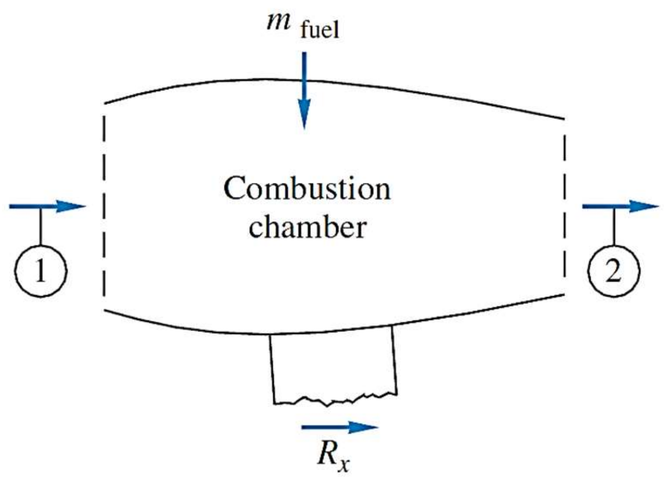
The fuel-to-air ratio is 1:30;

$$\therefore \dot{m}_2 = 150.6\left(1+\frac{1}{30}\right) = 155.6 \text{ kg/s}$$

Problem 5

from steady flow momentum principle:

$$\sum \vec{F}_{CV} = \int_{CS} \vec{\mathbf{V}} \rho \left(\vec{\mathbf{V}} \cdot \hat{\mathbf{n}} \right) dA$$



$$\rightarrow x : \sum F_x = \left(\dot{m} V_x \right)_{out} - \left(\dot{m} V_x \right)_{in}$$

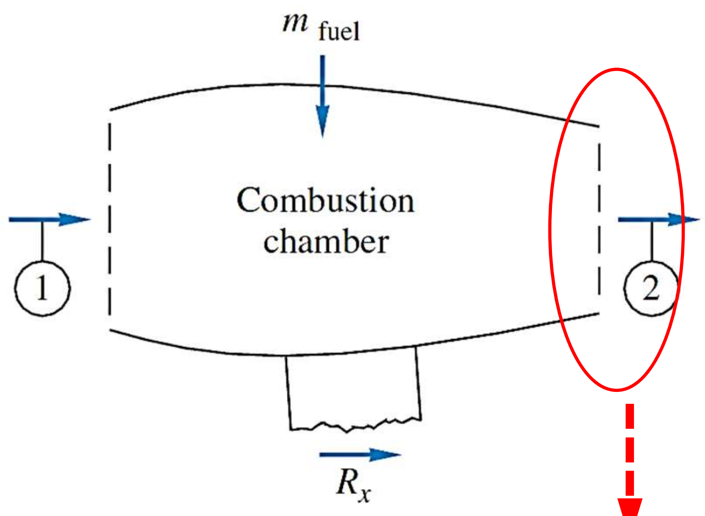
$$\Rightarrow R_x = \dot{m}_2 V_{2x} - \left(\dot{m}_1 V_{1x} + \dot{m}_{fuel} V_{f,x} \right)$$

$$\Rightarrow R_x = (155.6)(900) - \left((150.6)(250) + \dot{m}_{fuel}(0) \right)$$

$$\therefore \quad R_x = 102.4 \text{ kN} \qquad \text{(to right)} \qquad \text{Ans.}$$

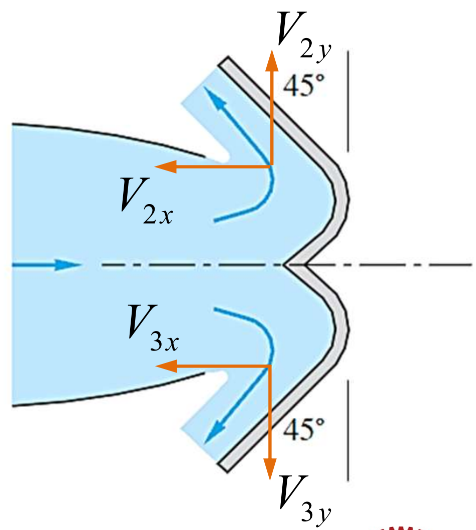
Problem 5

(b) Suppose that a deflector is deployed at the exit of the jet engine of Prob. 5(a), as shown in Figure. What will the reaction **R_x** on the test stand be now?



from steady flow momentum principle:

$$\sum \vec{F}_{CV} = \int_{CS} \vec{\mathbf{V}} \rho \left(\vec{\mathbf{V}} \cdot \hat{\mathbf{n}} \right) dA$$



$$\rightarrow x : \sum F_x = \left(\dot{m} V_x \right)_{out} - \left(\dot{m} V_x \right)_{in}$$

$$\Rightarrow R_x = \left(\dot{m}_2 V_{2x} + \dot{m}_3 V_{3x} \right) - \left(\dot{m}_1 V_{1x} + \dot{m}_{fuel} V_{f,x} \right)$$

$$\Rightarrow R_x = \left(\left(\frac{155.6}{2} \right) (-900 \cos 45°) + \left(\frac{155.6}{2} \right) (-900 \cos 45°) \right) - \left((150.6)(250) + \dot{m}_{fuel}(0) \right)$$

$$\Rightarrow R_x = -137.6 \text{ kN}$$

$$\therefore \quad R_x = 137.6 \text{ kN} \quad \text{(to left)} \qquad \text{Ans. (b)}$$